# D-Brane Approach to Black Hole Quantum Mechanics

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### Abstract

Strominger and Vafa have used D-brane technology to identify and precisely count the degenerate quantum states responsible for the entropy of certain extremal, BPS-saturated black holes. Here we give a Type-II D-brane description of a class of extremal and non-extremal five-dimensional Reissner-Nordström solutions and identify a corresponding set of degenerate D-brane configurations. We use this information to do a string theory calculation of the entropy, radiation rate and "Hawking" temperature. The results agree perfectly with standard Hawking results for the corresponding nearly extremal Reissner-Nordström black holes. Although these calculations suffer from open-string strong coupling problems, we give some reasons to believe that they are nonetheless qualitatively reliable. In this optimistic scenario there would be no "information loss" in black hole quantum evolution.

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### 1. Introduction and Motivation

Black hole thermodynamics [1,2] implies that black holes have an entropy proportional to the area of the event horizon:  $S = A/4G_N$ , where  $G_N$  is the Newton constant and  $\hbar = 1$ . Associated with this entropy are apparent problems of unitarity and information loss in the quantum dynamics of the black hole [3]. It has long been felt that the resolution of the paradoxes must lie in a microscopic quantum description of the states responsible for the entropy and that string theory, the only consistent quantum gravity, is the right framework in which to look for such a description [4,5,6]. Progress along this line was, however, held up for some time by the absence of sufficiently explicit string theory descriptions of extended spacetime objects like macroscopic black holes. This obstacle has recently been rendered less daunting by the advent of D-branes [7], which give a simple description of stringy solitons in terms of open strings with unusual boundary conditions.

Most D-branes are not black holes in any useful sense, but a few examples which are have now been found and the problem of counting states has begun to receive attention. In particular, Strominger and Vafa [8] have found a D-brane description of an extremal, BPS-saturated, Reissner-Nordström black hole in type-II string theory compactified to five dimensions on  $S_1 \times K_3$  and shown for the first time that state counting reproduces the Bekenstein-Hawking entropy. Surprisingly detailed features of the emerging picture of black hole entropy had actually been anticipated in work based on rather general string theory considerations [9], and it now seems that some of the same features can be extracted from a more "conventional" conformal field theory approach [10].

Nonetheless, the D-brane approach represents a major advance in our quantitative understanding of these matters and, in this paper, we will use it to study the physics of a class of nearly extremal five-dimensional Reissner-Nordström black holes. The extremal case is very similar to that studied in [8], but the geometric picture used here, based on toroidal compactification, seems particularly well suited to discussing the non-extremal case. We work in the type IIB string theory on  $M^5 \times T^5$  and we construct a D-brane configuration, preserving 1/8 of the supersymmetries, such that the corresponding supergravity solutions describe black holes with finite horizon area in five dimensions. This configuration consists of a large number of 5D-branes wrapped on  $T^5$ , plus 1D-branes wrapped along one of the compact directions, plus an ensemble of open strings with one end attached to a 1D-brane and the other to a 5D-brane. All the open strings move in the same direction so that the solution carries a large total momentum in this internal dimension.

Following [8], we derive the area law for the entropy of the extremal black hole by state counting. Then we consider "nearly" extremal configurations whose deviations from extremality are small in the macroscopic sense ( $\delta M/M \ll 1$ ) but large in the microscopic sense ( $\delta M \gg$  masses of low energy excitations). A D-brane calculation of entropy gives exactly the same results as the canonical calculation based on the associated Reissner-Nordström metric [2,11] (at least in the leading nearly extremal limit). Hawking radiation is seen as a simple decay process with a rate that could be calculated using perturbative string techniques. This rate has thermal properties with exactly the Hawking temperature [1]. The overall coefficient is proportional to the area.

A skeptic could object to our naive perturbative calculation on the grounds that the open strings attached to a macroscopic horizon are inevitably strongly coupled. We will present qualitative arguments why the classical black hole description and the perturbative D-brane picture could be valid at the same time, thus rationalizing our treatment of non-extremal black hole physics. In this optimistic scenario, quantum evolution of the D-brane system would be manifestly unitary and there would be no black hole information loss.

An essential issue, which is brought to the fore by our treatment, is the manner in which the states responsible for the black hole entropy encode, as they must, the state of particles which have fallen through the macroscopic horizon. If this can be done, it may give give concrete shape to the imaginative "holographic" ideas about horizon dynamics that have been proposed by Susskind [5,6].

#### 2. Classical Black Hole Solutions in 5 Dimensions.

The five dimensional Reissner-Nordström black hole is a solution of the five dimensional Einstein plus Maxwell action. The metric reads [11]

$$ds^{2} = -\lambda dt^{2} + \lambda^{-1} dr^{2} + r^{2} d\Omega_{3}^{2}$$
(2.1)

$$\lambda = \left(1 - \frac{r_+^2}{r^2}\right) \left(1 - \frac{r_-^2}{r^2}\right)$$

There is a horizon at  $r = r_+$ , mass and charge are given by  $\dagger$ 

$$M = \frac{3\pi}{8G_N}(r_+^2 + r_-^2) \qquad Q = r_+ r_-$$
 (2.2)

 $<sup>^{\</sup>dagger}$  Here, and in the rest of the paper,  $G_N$  denotes the five dimensional Newton constant.

and the entropy is proportional to the area of the horizon,  $S = A/4G_N = \pi^2 r_+^3/2G_N$ . The extremal solution is obtained by taking  $r_+ = r_- \equiv r_e$  and has zero temperature but non-zero entropy. If we take a nearly extremal solution  $(r_+ \sim r_-)$ , keeping the charge fixed, we find that the entropy behaves as

$$\frac{\Delta S}{S_e} = \frac{3}{\sqrt{2}} \sqrt{\frac{\Delta M}{M_e}} \tag{2.3}$$

where  $M_e$ ,  $S_e$  are the extremal values of the mass and entropy respectively. The Hawking temperature for such a nearly extremal black hole is

$$T_H = \frac{2}{\pi r_e} \sqrt{\frac{\Delta M}{2M_e}} \tag{2.4}$$

Now let us show that there is a configuration of strings and solitons in the type IIB string theory on  $M^5 \times T^5$  that can be be identified with the Reissner-Nordström black hole. We denote the  $M^5$  coordinates by  $(t, x_1, x_2, x_3, x_4)$  and the torus coordinates by  $(x_5, x_6, x_7, x_8, x_9)$ . The configuration we study is constructed as follows: first wrap  $Q_5$  5D-branes on the torus, then wrap  $Q_1$  1D-branes along an  $S_1$  of the torus (to be definite, let us say that it is the direction  $\hat{5}$ ). This leaves  $1/2 \times 1/2 = 1/4$  of the supersymmetries unbroken. To see this, notice that in the ten dimensional type IIB theory the supersymmetries were generated by two independent chiral spinors  $\epsilon_R$  and  $\epsilon_L$  (  $\Gamma^{11}\epsilon_{R,L} = \epsilon_{R,L}$ ). The presence of the strings and the fivebranes imposes additional conditions on the surviving supersymmetries

$$\epsilon_R = \Gamma^0 \Gamma^5 \epsilon_L \qquad \epsilon_R = \Gamma^0 \Gamma^5 \Gamma^6 \Gamma^7 \Gamma^8 \Gamma^9 \epsilon_L$$
(2.5)

If, in addition, we put some left-moving momentum along the  $\hat{5}$  direction then we further break the supersymmetry to 1/8 of the original, through the extra condition

$$\Gamma^0 \Gamma^5 \epsilon_R = \epsilon_R. \tag{2.6}$$

Taken together, this gives the following decomposition for the surviving spinor

$$\epsilon_L = \epsilon_R = \epsilon_{SO(1,1)}^+ \epsilon_{SO(4)}^+ \epsilon_{SO(4)}^+ \tag{2.7}$$

The positive chirality SO(4) spinor is pseudoreal and has two independent components so that 4 out of the original 32 supersymmetries are preserved by this configuration. Now we want to find a classical supergravity solution associated to this 1-brane plus 5-brane plus momentum configuration. Fortunately, Tseytlin [10] has found a closely related classical

solution in type II string theory on  $M^5 \times T^5$ . His configuration consists of solitonic five branes and fundamental strings, and carries momentum along the direction  $\hat{5}$ . In order to find a D-brane description we use the strong-weak coupling duality of this theory [12,13]. Such a duality transformation turns fundamental strings into 1D-branes and solitonic 5branes into 5D-branes and turns Tseytlin's solution into just what we are looking for.

The dilaton field and the ten dimensional string metric emerging from this procedure are

$$e^{-2\phi_{10}} = f_5 f_1^{-1}$$

$$ds_{str}^2 = f_1^{-\frac{1}{2}} f_5^{-\frac{1}{2}} \left( -dt^2 + dx_5^2 + K(dt - dx_5)^2 \right) + f_1^{\frac{1}{2}} f_5^{\frac{1}{2}} (dx_1^2 + \dots + dx_4^2) + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}} (dx_6^2 + \dots + dx_9^2).$$
(2.8)

The solution contains three harmonic functions of the transverse coordinates  $(x^2 = x_1^2 + \cdots + x_4^2)$  associated with the three charges needed to define the solution:

$$f_1 = 1 + \frac{c_1 Q_1}{x^2}$$
  $f_5 = 1 + \frac{c_5 Q_5}{x^2}$   $K = \frac{c_p N}{x^2}$  (2.9)

Some components of the Ramond-Ramond antisymmetric tensor field,  $B_{\mu\nu}^{RR}$ , are also excited and they behave as gauge fields when we dimensionally reduce to five dimensions. The three independent charges arise as follows:  $Q_1$  is a RR electric charge, coming from  $B_{05}^{RR}$  and counts the 1D-branes.  $Q_5$  is a magnetic charge for the three form field strength  $H_3^{RR} = dB_2^{RR}$ , which is dual in five dimensions to a gauge field,  $H_3^{RR} = *F_2$ .  $Q_5$  is thus an electric charge for the gauge field  $F_2$  and it counts the 5D-branes. The third charge, N, corresponds to the total momentum carried by the open strings traveling on the branes in the direction  $\hat{5}$ , and it is associated to the five dimensional Kaluza-Klein gauge field coming from the  $G_{50}$  component of the metric. The coefficients in (2.9),

$$c_1 = \frac{4G_N R}{g\pi\alpha'} \qquad c_5 = \alpha' g \qquad c_p = \frac{4G_N}{\pi R}, \qquad (2.10)$$

are defined so that the charges  $Q_1$ ,  $Q_5$  and N are naturally integer quantized. In these expressions, g is the ten dimensional string coupling constant, defined so that S-duality takes  $g \to 1/g$  (note that  $G_N \sim g^{2\dagger}$ ). The precise value of these coefficients follows from matching the field of a fundamental string to its source following [14] and from the solitonic fivebrane Dirac quantization for the charge [15]. Note that the total momentum in the direction  $\hat{5}$  is quantized as N/R, where R is the radius of the circle.

 $<sup>^{\</sup>dagger}$  The precise relation between g and  $G_N$  will not be need in what follows.

Now we dimensionally reduce (2.8) to five dimensions in order to read off black hole properties. The standard method of [16] yields a five-dimensional Einstein metric,  $g_E^5 = e^{-4\phi_5/3}G_{string}^5$ ,

$$ds_E^2 = -\frac{1}{(f_1 f_5 (1+K))^{\frac{2}{3}}} dt^2 + (f_1 f_5 (1+K))^{\frac{1}{3}} (dx_1^2 + \dots + dx_4^2)$$
 (2.11)

which can be interpreted as a five dimensional extremal, charged, supersymmetric black hole with nonzero horizon area. Calculating the horizon area in this metric (2.11) we get the entropy

$$S_e = \frac{A_H}{4G_N} = 2\pi \sqrt{NQ_1 Q_5} \tag{2.12}$$

In this form the entropy does not depend on any of the continuous parameters like the coupling constant or the sizes of the internal circles, etc. This "topological" character of the entropy was emphasized in [17,9,10]. It is also symmetric under interchange of  $N, Q_1, Q_5$ . In fact, U duality [13,18,19] interchanges the three charges. To show it in a more specific fashion, let us define  $T_i$  to be the usual T-duality that inverts the compactification radius in the direction i and S the ten dimensional S duality of type IIB theory. Then a transformation that sends  $(N, Q_1, Q_5)$  to  $(Q_1, Q_5, N)$  is  $U=T_9T_8T_7T_6ST_6T_5$ . If the ten dimensional coupling constant is small and the radii are of order  $\alpha'$  it leaves the ten dimensional and the 5 dimensional coupling constants small but it modifies the radii of the compact dimensions. This symmetry of the string theory will be crucially used below.

The standard five-dimensional extremal Reissner-Nordström solution (2.1) is recovered when the charges are chosen such that

$$c_p N = c_1 Q_1 = c_5 Q_5 = r_e^2 (2.13)$$

and the coordinate transformation  $r^2 = x^2 + r_e^2$  is applied. The crucial point is that, for this ratio of charges, the dilaton field and the internal compactification geometry are independent of position and the distinction between the ten-dimensional and five-dimensional geometries evaporates. What is at issue is not so much the charges as the different types of energy-momentum densities with which they are associated. An intuitive picture of what goes on is this: a p-brane produces a dilaton field of the form  $e^{-2\phi_{10}} = f_p^{\frac{p-3}{2}}$ , with  $f_p$  a harmonic function [20]. A superposition of branes produces a product of such functions and one sees how 1-branes can cancel 5-branes in their effect on the dilaton. A similar thing is true for the compactification volume: For any p-brane, the string metric is such

that the volume parallel to the brane shrinks and the volume perpendicular to it expands as we get closer to the brane. It is easy to see how superposing 1-branes and 5-branes can stabilize the volume in the directions  $\hat{6}$ ,  $\hat{7}$ ,  $\hat{8}$ ,  $\hat{9}$ , since it is perpendicular to the 1-brane and parallel to the 5-brane. The volume in the direction  $\hat{5}$  would still seem to shrink, due to the tension of the branes. This is indeed why we put momentum along the 1-branes, to balance the tension and produce a stable radius in the  $\hat{5}$  direction. If we balance the charges precisely (2.13) (we can always do this for large charges) the moduli scalar fields associated with the compactified dimensions are not excited at all, which is what we need to get the Reissner-Nordström black hole.

### 3. D-Brane Description of Extremal Black Holes

Now we want to give a D-brane description of this black hole<sup>†</sup>. We consider type IIB string theory compactified on  $M^5 \times T^5$ . As we know, type IIB has 2p + 1-brane soliton solutions which have a D-brane description via open strings with Dirichlet boundary conditions. In particular, there are both 1-branes and 5-branes and we can use them to construct a multi-brane state in the same spirit as the construction of 5D Reissner-Nordström black holes in the previous section. To be precise, we superpose  $Q_5$  5D-branes wrapping around the  $T^5$  and  $Q_1$  1D-branes wrapping around one of the compact directions (call it  $\hat{5}$ ). The supersymmetry analysis is the same as for the classical solution and the branes with no excitations break 1/4 of the supersymmetries. These membranes can have excitations and we will be interested in the excitations that break just one additional supersymmetry by imposing the extra condition (2.6). This rules out the massive open strings attached to the D-branes, so we concentrate then only on the massless excitations of the open strings that live on the D-brane. Actually (2.6) implies that the open strings have to move in the direction  $\hat{5}$  and be left moving. Momentum along the direction  $\hat{5}$  is quantized in units of 1/R. If we consider a state containing  $n_i$  open strings with momentum i (all momenta of the same sign!), the total momentum in the direction  $\hat{5}$  is

$$P = \frac{1}{R} \sum_{i} i n_i = \frac{N}{R},$$

an expression with the structure of a string level number (this fact was exploited in a related context in [22] to count BPS and non-BPS states of oscillating D-strings).

 $<sup>^{\</sup>dagger}~$  For a discussion of a two dimensional black hole using D-branes see [21].

We will first count, via a straightforward (modulo one subtlety) denumeration of the open string states supported by this collection of branes, the degeneracy of the states corresponding to extremal black holes. Our result for this will, in the end, not be significantly different from that of [8], which is based on a sophisticated analysis of the cohomology of instanton moduli spaces [23]. We present our own method anyway, if only because we find it easier to generalize to the counting of non-extremal states, the ultimate topic of this paper. There are many types of open strings to consider: those that go from one 1-brane to another 1-brane, which we denote as (1,1) strings, as well as the corresponding (5,5), (1,5) and (5,1) strings (the last two being different because the strings are oriented). We want to construct the BPS states that have the highest degeneracy and that corresponds to taking the strings all of (1,5) or (5,1) type. In fact, it was shown in [19] that, in the low energy effective world volume field theory, these states are charged matter fields in the fundamental representation of the  $U(Q_1)\times U(Q_5)$  gauge theory generated by the (1,1) and (5,5) strings (see also [24]). The presence of many open strings effectively gives an expectation value to these fields, which then act as Higgs fields and break the U(Q) symmetries. In short, if we excite many (1,5) or (5,1) strings the (1,1) and (5,5) strings become massive and can be dropped from the state counting. This also indicates that the configurations under discussion are really bound states, since the transverse motion of the branes relative to each other is brought about by (1,1) or (5,5) string excitations.

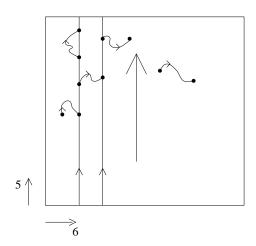


FIGURE 1: Configuration of intersecting D-branes. We show two of the internal dimensions and several types of open strings. The open strings going between 1 and 5 branes are the most relevant for the black holes that we analyze.

Let us consider then the (1,5) and (5,1) strings. We have 2 bosons with NN boundary conditions, 4 with ND and 4 with DD. The vacuum energy of the bosons is then E =4(-1/24+1/48). Of the fermions in the NS state, the 4 that are in the ND directions will end up having R-type quantization conditions. The net fermionic vacuum energy is E = 4(1/24 - 1/48) and exactly cancels the bosonic one. This vacuum is a spinor under SO(4), is acted on by  $\Gamma^6, \Gamma^7, \Gamma^8, \Gamma^9$  and obeys the GSO chirality condition  $\Gamma^6\Gamma^7\Gamma^8\Gamma^9\chi = \chi$ . What remains is a two dimensional representation. There are two possible orientations and they can be attached to any of the different branes of each type. This gives a total of  $4Q_1Q_5$  different possible states for the string. Now consider the Ramond sector. The four internal fermions transverse to the string will have NS type boundary conditions. The vacuum again has zero energy and is an SO(1,5) spinor and thus a fermion. Again the GSO condition implies that only the positive chirality representation of SO(1,5) survives. It also has to be left moving so that only the  $2^+_+$  under  $SO(1,1)\times SO(4)$  survives. This gives the same number of states as for the bosons. In summary, for each momentum we could have  $4Q_1Q_5$  bosons and  $4Q_1Q_5$  fermions. Roughly speaking, the state counting is the same as that of the left moving oscillator modes of  $4Q_1Q_5$  free superconformal fields. The asymptotic formula for the degeneracy of level N states in a conformal field theory of central charge c is  $d(N) \sim e^{2\pi\sqrt{cN/6}}$ . Including both the fermions and the bosons, we have  $c = 4Q_1Q_5 \times 3/2$  leading to an entropy

$$S_e = 2\pi \sqrt{Q_1 Q_5 N}$$
, (3.1)

in perfect agreement with (2.12). This argument is not quite right since it treats the multiple 1-branes and 5-branes as distinguishable. Since they are at least separately indistinguishable, one might guess that the superconformal fields should be taken to describe the orbifold  $(T^4)^{Q_1Q_5}/S(Q_1)S(Q_5)$ , where S(Q) is the permutation group on Q items. A deeper analysis, based on the instanton moduli space approach [23], strongly suggests the correct answer is  $(T^4)^{Q_1Q_5}/S(Q_1Q_5)$  as if, somehow, the 1-branes were effectively indistinguishable from the 5-branes as well as themselves. We would like to understand this point better<sup>†</sup> but, fortunately for us, the various orbifolds have the same central charge and the same large-N behavior of the degeneracy of states.

 $<sup>^{\</sup>dagger}$  We are grateful to Cumrum Vafa for pointing out an error in our original treatment of this point.

It is interesting to note that we could have considered an analogous black hole in the heterotic string theory on  $M^5 \times T^5$  built out of fundamental strings and solitonic fivebranes. These are the dyonic black holes considered by various authors [9,10,25,26]. In that case, the analogous D-brane description takes place in the type I theory, which indeed has 1D-branes and 5D-branes corresponding to the fundamental string and the solitonic fivebrane [27]. The D-brane counting can also be done, and it is interesting to notice that to get the correct result one must include the 5D-brane SU(2) degrees of freedom found in [28].

Something remarkable has happened here. We started with some configuration of D-branes sitting at r=0, a point in 5-dimensional space. To start with, this is a "point with nothing inside it." However, having put all these open strings on the branes we find that the configuration matches a solution of the classical low energy action such that r=0 is really a 3-sphere with non-zero area! What happened? Well, the ten dimensional classical solutions for D-branes show that as we get closer to the D-brane the transverse space expands and the longitudinal space shrinks. This configuration has expanded the transverse space in such great way that what previously was a point is now a sphere. The most exciting aspect of this is that the classical solution continues beyond the horizon, into the black hole singularity, whereas, according to the D-brane picture space simply stops at r=0. States inside the horizon would have to be described by the massless modes on the D-brane. The basic horizon degrees of freedom are denumerated by three integers: the momentum, the index labeling the 1-brane and the index labeling the 5-brane. When a string "falls" into the black hole and crosses the horizon, it turns into a pair of open strings traveling on the D-branes (see figure 2). Since an infalling geodesic is labeled by three angles on the 3-sphere and the open string states are labeled by three integers (i,j,n)(with  $1 \leq i \leq Q_1$ ,  $1 \leq j \leq Q_5$  and n the momentum), there is at least a correspondence between numbers of degrees of freedom. Of course, the transformation of "physical" space coordinates to the three integers could be very complicated indeed. The existence of an upper bound on these integers could be related to the fact that there can be at most "one bit per unit area" [29]. All of this is reminiscent of the "holographic" principle [29], as well as the membrane paradigm [6,30], in that dynamics occurring inside the black hole would be described as occurring on the horizon.

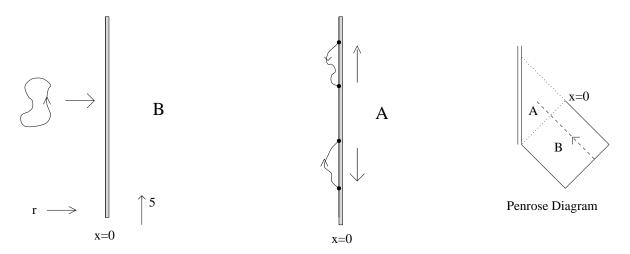


FIGURE 2: D-brane description of a string before and after falling through the horizon.

### 4. Non-Extremal Black D-Branes and Hawking Radiation.

We now turn to a discussion of nearly extremal five-dimensional black holes. We will find it convenient to restrict our attention to the special black holes in which all the scalar moduli fields are constant and the compactification geometry is completely passive. We saw that this could be achieved in the extremal case by imposing a certain relation (2.13) between the three types of charge. Perturbation theory assures us that we can achieve the same result (cancellation of the sources for all the compactification moduli) for slightly non-extremal solutions by imposing the same ratio on the total source stressenergies due to the three different charges (we will shortly explain this is more detail). Since the coupling constant in these solutions is uniform in space we can choose it to be weak everywhere. This should be a favorable case for examining the non-BPS states of the D-brane, doing perturbative computations of their interactions and comparing to the canonical expectations for the non-extremal Reissner-Nordström black hole. We will see that agreement between the two approaches is just as impressive as in the extremal case. There is, however, a hitch: the presence of a large number of D-branes  $(Q_1, Q_5 \gg$ 1) amplifies the effective open string coupling constant and, in principle, renders any perturbative analysis of horizon dynamics unreliable [8,31,32]. We think the situation may not be so desperate and will present some (non-rigorous!) arguments that open string loop corrections might not, in fact, change the essential physics.

We perturb away from the BPS limit in a macroscopically small but microscopically large fashion  $(M \gg \delta M \gg \text{mass of typical excitations})$ . There are many ways to do

this by adding stringy excitations to the basic D-brane configuration. We are interested in those excitations which cause the entropy to increase most rapidly with added energy. One could add fundamental string modes traveling on the torus, but they have too small a central charge to be relevant. Massive open or closed string excitations also give a subleading contribution since the entropy of a gas of these excitations increases at most as  $\delta M^{2/3}$ , and we will find a leading contribution going as  $\delta M^{1/2}$ . One could have five brane excitations in any direction, but that entropy increases as  $\delta M^{\frac{p}{p+1}}$  for a membrane of p dimensions. So we conclude that the most important will be the ones along the string. There are open string modes of type (1,1) and (5,5) starting and ending on branes of the same type, but these modes are massive when there is a condensate of open strings of type (1,5). The only ones left are the modes of type (1,5) and (5,1). If we perturb away from a purely left-moving extremal background by adding  $\delta N_R$  right-moving oscillations, we also have to add  $\delta N_L = \delta N_R$  left-moving oscillations to keep the total  $N = N_L - N_R$  charge fixed. These oscillations have 4 bosons and 4 fermions, so the central charge is the same as it was before. The change in left-moving entropy is proportional to  $\sqrt{N+\delta N_R}-\sqrt{N}$  and is of order  $\delta N_R/N_L$ . The change in right-moving entropy is, however, of order  $\sqrt{\delta N_R/N_L}$ and dominates. More specifically, we find that

$$\left. \frac{\Delta S}{S_e} \right|_{oscill} = \sqrt{\frac{\delta N_R}{N}} \; .$$

One would like to reexpress this entropy change in terms of the change in mass of the configuration and compare it with the  $\Delta S$  versus  $\Delta M$  relation that would be extracted from the geometry of an appropriate Reissner-Nordström solution. The question is, what solution? In our thinking about this problem we have chosen to focus on the strict five-dimensional Reissner-Nordström solutions, i.e. solutions for which the dilaton and the various compactification moduli are strictly constant. Indeed, the extremal solution, from which we start, is of this character because it satisfies (2.13). The role of that condition is just to arrange that the 1-brane, 5-brane and open string stresses in the compactified dimensions add up to something which is, for instance, isotropic in the internal directions. If, as above, we increase the total energy carried by oscillations moving in the  $\hat{5}$  direction and change nothing else, the  $\hat{5}$  direction is singled out, compactification moduli will be excited and the compactification geometry will vary from point to point.

There is nothing wrong with this, but we would like to stay within the framework of the strict five-dimensional Reissner-Nordström solution (2.1). Using the qualitative arguments

given at the end of Sec. 2, it seems that to do this one should increase the source stresses due to the 1-branes and 5-branes in the same proportion as one has increased the energy of the open strings by introducing right-movers. Thus we should augment the original "branes" wound around the internal directions by an appropriate number of "anti-branes" (for both 1-branes and 5-branes). Here the word "anti" means having the opposite RR charge and opposite orientation. The anti-branes can eventually find the branes and annihilate, but left-movers can also annihilate right-movers, and it seems to us that denumeration of states is equally meaningful in the two cases. We need to add equal amounts of branes and antibranes to keep the total Ramond-Ramond charges fixed. The precise quantity one should add is determined, in perturbation theory, by the same reasoning that determined the relation between the charges (2.13). The logic behind that relation was that the condition that the dilaton not be excited fixes the relation of the anti-5-branes to the anti-1-branes, while the condition that the size of the direction  $\hat{5}$  does not change fixes the relation between the branes and the oscillators. Calling  $\delta Q_1^A, \delta Q_5^A$  the extra amount of anti 1-and-5-branes that must be added to balance an increment  $\delta N_R$ , the relation which must lead to a strict five-dimensional Reissner-Nordström solution is

$$c_p \delta N_R = c_1 \delta Q_1^A = c_5 \delta Q_5^A. \tag{4.1}$$

We are assuming that the total added mass  $\delta M$  is much bigger than the mass of a single D-brane.

Just as there is an entropy increment associated with the various ways to achieve  $\delta N_R$ , there are entropy increments associated with  $\delta Q_1^A$  and  $\delta Q_5^A$ . They are presumably independent and should be added to get the total entropy increment. We have already calculated the increase due to the right and left movers, but it is not obvious what the entropy increase due to the anti-branes will be. There is however a U-duality [13,18,23,19] that, for example, turns anti-1-branes into right moving momentum states at the price of some transformation of coupling and compactification radii. Since the entropy increase is independent of the coupling constant and the compactification radii, we will take the duality argument as telling us that the counting of the brane-antibrane states is just the same as the counting of the left- and right-moving oscillator states. The net result for the entropy increment is

$$\frac{\Delta S}{S} \bigg|_{anti-1-branes} = \sqrt{\frac{\delta Q_1^A}{Q_1}}.$$

Since the same argument applies to  $\delta Q_5^A$ , the total entropy of the non-extremal solution is

$$\frac{\Delta S}{S}\Big|_{total} = \sqrt{\frac{\delta N_R}{N}} + \sqrt{\frac{\delta Q_1^A}{Q_1}} + \sqrt{\frac{\delta Q_5^A}{Q_5}} = 3\sqrt{\frac{\delta N_R}{N}}$$
(4.2)

since all ratios are equal by (4.1)(2.13). It is more to the point to express this result in terms of the mass. For the extremal black hole, the mass is just the sum of the charges (with appropriate coefficients). When we go slightly away from extremality by adding "anticharges", a simple superposition principle (valid to lowest order in g) gives

$$M = \frac{1}{R} \left\{ \frac{c_1}{c_p} (Q_1 + 2\delta Q_1^A) + \frac{c_5}{c_p} (Q_5 + 2\delta Q_5^A) + (N + 2\delta N_R) \right\} .$$

where  $\delta N_R$  represents the contribution of the added right-movers to the mass and  $N + \delta N_R$  represents the contribution of the new total number of left-movers and so on for the other two types of charge. Because of the "balance" conditions on N,  $\delta N_R$  (2.13), (4.1), we have  $\delta N_R/N = \delta M/2M$  and, finally,

$$\left. \frac{\delta S_e}{S} \right|_{string} = \frac{3}{\sqrt{2}} \sqrt{\frac{\delta M}{M_e}}$$

This is the standard Bekenstein-Hawking result for the strict five-dimensional Reissner-Nordström black hole, with the correct normalization and functional dependence on mass. Although the arguments that led us here are less than rigorous (especially the duality argument for entropies associated with branes and antibranes), the simple end result gives us some confidence in the intermediate steps. If we had studied the deviation from extremality arising from adding only the right-moving momentum states (thus moving away from a strict five-dimensional solution), the entropy increment formula would simply have been missing the factor of three, presumably the correct result for that state [31].

These non-BPS states will decay. The simplest decay process is a collision of a right moving string excitation with a left moving one to give a closed string that leaves the brane. We will calculate the emission rate for chargeless particles, so that the basic process is a right moving open string with momentum  $p_5 = n/R$  colliding with a left moving one of momentum  $p_5 = -n/R$  to give a closed string of energy  $k_0 = 2n/R$ . We will calculate the rate for this process according to the usual rules of relativistic quantum mechanics and show that the radiation has a thermal spectrum if we do not know the initial microscopic state of the black hole.

The state of the black hole is specified by giving the left and right moving occupation numbers of each of the  $4Q_1Q_5$  bosonic and fermionic oscillators. In fact, the nearly extremal black holes live in a subsector of the total Hilbert space that is isomorphic to the Hilbert space of a one dimensional gas of massless particles of  $4Q_1Q_5$  different types, either bosons or fermions (up to subtleties related to the orbifolding procedure discussed above). This state  $|\Psi_i\rangle$  can then emit a closed string and become  $|\Psi_f\rangle$ . The rate, averaged over initial states and summed over final states, as one would do for calculating the decay rate of an unpolarized atom, is

$$d\Gamma \sim \frac{d^4k}{k_0} \frac{1}{p_0^R p_0^L} \delta(k_0 - (p_0^R + p_0^L)) \sum_{i,f} |\langle \Psi_f | H_{int} | \Psi_i \rangle|^2$$

The relevant string amplitude for this process is given by a correlation function on the disc with two insertions on the boundary, corresponding to the two open string states and an insertion in the interior, corresponding to the closed string state (in the spirit of [33]). The boundary vertex operators change boundary conditions for four of the coordinates. We consider the case when the outgoing closed string is a spin zero boson in five dimensions, so that it corresponds to the dilaton, the internal metric, internal  $B_{\mu\nu}$  fields, or internal components of RR gauge fields. This disc amplitude, call it  $\mathcal{A}$ , is proportional to the string coupling constant g and it is basically the same amplitude that that would appear if we were to calculate the absorption cross section of the black hole.

Note that performing the average over initial and sum over final states will just produce a factor of the form  $\rho_L(n)\rho_R(n)$  with

$$\rho_R(n) = \frac{1}{N_i} \sum_i \langle \Psi_i | a_n^{R\dagger} a_n^R | \Psi_i \rangle \tag{4.3}$$

where  $N_i$  is the total number of initial states and  $a_n^R$  is the creation operator for one of the  $4Q_1Q_4$  bosonic open string states. The factor  $\rho_L(n)$  is similar. Since we are just averaging over all possible initial states with given value of  $\delta N_R$ , this corresponds to taking the expectation value of  $a_n^{\dagger}a_n$  in the microcanonical ensemble with total energy  $\delta N_R$  of a one dimensional gas. Because  $\delta N_R$  is large compared to one (but still much smaller than  $N_L$ ), we can calculate (4.3) in the canonical ensemble. Writing the partition function as

$$Z = \sum_{N} q^{N} d(N) = \sum_{N} q^{N} e^{2\pi \sqrt{Q_{1}Q_{5}N}},$$

doing a saddle point evaluation and then determining q from

$$\delta N_R = q \frac{\partial}{\partial q} \log Z,$$

we find  $\log q = -\pi \sqrt{Q_1 Q_5/\delta N_R}$ . Then we can calculate the occupation number of that level as

$$\rho_R(k_0) = \frac{q^n}{1 - q^n} = \frac{e^{\frac{-k_0}{T}}}{1 - e^{\frac{-k_0}{T}}}$$

We can read off the "right moving" temperature

$$T_R = \frac{2}{\pi} \frac{1}{R} \sqrt{\frac{\delta N_R}{Q_1 Q_5}}$$

Now using (3.1), (2.10) and (2.13) we get

$$T_R = \frac{2}{\pi r_e} \sqrt{\frac{\delta M}{2M}} \tag{4.4}$$

There is a similar factor for the left movers  $\rho_L$  with a similar looking temperature

$$T_L = \frac{2}{\pi} \frac{1}{R} \sqrt{\frac{N}{Q_1 Q_5}}$$

Since  $T_R \ll T_L$  the typical energy of the outgoing string will be  $k_0 \sim T_R$  and  $k_0/T_L \sim T_R/T_L \ll 1$  so that we could approximate

$$\rho_L \sim \frac{T_L}{k_0} = \frac{2}{\pi k_0 R} \sqrt{\frac{N}{Q_1 Q_5}} \tag{4.5}$$

The expression for the rate then is, up to a numerical constant

$$d\Gamma \sim \frac{d^4k}{k_0} \frac{1}{p_0^R p_0^L} |\mathcal{A}|^2 Q_1 Q_5 \rho_R(k_0) \rho_L(k_0)$$
(4.6)

where  $\mathcal{A}$  is the disc diagram result. The factor  $Q_1Q_5$  is due to the fact that we have this many distinct open string levels for each momentum n. The final expression for the rate is then, using (4.5) in (4.6)

$$d\Gamma \sim (\text{Area}) \frac{e^{\frac{-k_0}{T_R}}}{1 - e^{\frac{-k_0}{T_R}}} d^4 k \tag{4.7}$$

We conclude that the emission is thermal, with a physical Hawking temperature

$$T_H = T_R = \frac{2}{\pi r_e} \sqrt{\frac{\delta M}{2M}} \tag{4.8}$$

which exactly matches the classical result (2.4). It is an interesting result that the area appeared correctly in (4.7). Actually, the coupling constant coming from the string amplitude  $\mathcal{A}$  is hidden in the expression for the area. Of course, it will be very interesting to calculate the coefficient in (4.7) to see whether it exactly matches the absorption coefficient of a large classical black hole. In fact the result should not depend on the internal polarizations of the outgoing particles. Note that U-duality [13] suggests that the antibrane excitations will also produce Hawking radiation with the same temperature (4.8). Notice that if we were emitting a spacetime fermion then the left moving string could be a boson and the right moving string a fermion, this produces the correct thermal factor for a spacetime fermion. The opposite possibility gives a much lower rate, since we do not have the enhancement due to the large  $\rho_L$  (4.5). Notice also that when separation from extremality is very small, then the number of right movers is small and the statistical arguments used to derive (4.7) fail. Such deviations were observed in [34].

We now examine the range of validity of these approximations. For the purposes of this argument, we take the compactification radii to be of order  $\alpha'$  and set  $\alpha' = 1$ . In this case (2.13) implies  $Q_1 \sim Q_5 \equiv Q$  and  $Q \sim gN$ . Then, by (2.12), we find that the area of the horizon is  $A \sim (g^2 N)^{3/2}$ . In order for the classical black hole interpretation to hold, this area has to be much larger than  $\alpha'$ , so  $g^2N \gg 1$ . Since we want g to be small, N is very large. This seems to invalidate the perturbative D-brane picture since open string loop corrections are of order  $gQ = g^2N$ , and, due to the large number Q of D-branes, they are likely to be large [8,31,32]. We will try to argue that open string corrections might in fact be suppressed. Let us first give a very qualitative argument. It has been noted [29] that string perturbation theory is expected to break down when the number of strings per unit volume is of order  $1/g^2$ . In this case we have of the order N open strings sitting at x = 0, but the area of this "point" has been expanded so that now the density of strings is  $N/A_H = N/(g^2N)^{3/2} \ll 1/g^2$  in the classical black hole domain  $(g^2N \gg 1)$ . Going back to the open string loops, we note that the loop will be in a non-trivial background of open strings. In fact, this background was crucial to obtain a small five dimensional coupling constant and non-zero area, which implies that somehow the D-branes might be "separated" from each other. We suspect that this background of open strings suppresses open string loops, enabling us to get results off extremality. This is of course something to be checked in detail. It is clear, however, that there are some circumstances where open string backgrounds suppress loop contributions. For example, compare loop contributions of n D-branes on top of each other and n widely separated D-branes. The difference is just a background of open string translational zero modes.

Finally, the fact that the perturbative D-brane treatment of non-extremal physics gives the right results strongly suggests that there is more than a grain of truth here. We think it quite possible that open string quantum corrections are not as large as suggested by naive estimates. The skeptic is entitled to disagree!

## 5. Summary, Conclusions and Speculations.

Our goal was to find a black hole which could simultaneously be described as a solution of low-energy effective field theory (so that the usual results of general relativity would be applicable) and as a weakly coupled D-brane soliton (so that perturbative string theory could be used to count states and evaluate entropies). Such an object would make possible a serious confrontation between string theory and the paradoxes of black hole quantum mechanics. We come close to what we need in certain solutions to ten-dimensional supergravity, compactified to five dimensions by wrapping 5D-branes around a  $T^5$  combined with 1D-branes winding in one of the  $T^5$  directions and and a gas of open strings carrying momentum. This object has three charges: the charge of the 5D-branes, the charge of the 1D-brane and the longitudinal momentum along the direction of the 1D-brane. When all three are excited, there is a horizon and nothing singular happens either to the dilaton or to the volume of the  $T^5$  at the horizon: from the point of view of low-energy physics everywhere outside the horizon, this is a large, classical five-dimensional Reissner-Nordström black hole. For appropriate choices of parameters, it can be made extremal and BPSsaturated (i.e. it preserves some supersymmetries). This solution has an event horizon with finite area, and therefore is expected to have an entropy, but it is hard to give a classical description of the states responsible for the entropy.

In the D-brane picture these states are easily denumerated and they lead to an entropy which agrees perfectly with that calculated from the horizon area of the corresponding supergravity solution. It is easy to construct a non-BPS saturated state by mixing in some right-moving open strings with the left-movers: these states are also denumerable and the increase of the entropy as the mass is increased above the extremal value can be computed.

There are other states, arising from mixing in one-branes and five-branes with opposite winding from the background branes, but U-duality [13] suggests how to count these states (how to do this state counting would be a total mystery without D-branes). The non-extremal entropy can be accounted for and the decay rate shows a thermal distribution with the "Hawking" temperature, in perfect agreement with the canonical formulae [2]. The precise mechanism by which non-extremal D-branes decay is apparent: two open strings of opposite longitudinal momentum annihilate to create a massless closed string state which escapes. The resulting radiation rate is proportional to the horizon area, but it has yet to be demonstrated that the overall numerical coefficient is that of a black body for all channels. For non-extremal black holes, these pleasing results are undercut by the fact that the large number of D-branes seem to render the horizon open strings strongly coupled. We gave qualitative arguments that the open string loops should in fact be suppressed, but they need to be developed further to be convincing.

What does all this mean? Assuming that loop corrections are under control, it means that unitary evolution of quantum states is *not* violated by this type of black hole. Entropy would be just the degeneracy of an identifiable set of states that live on the horizon. Thermal emission would be due to the lack of knowledge of the initial state, just as in real-world thermodynamics. There would be no information loss: quantum states falling into the black hole from the outside would cause a unitary evolution in the Hilbert space of horizon states that "records" the infalling quantum information. The remarkable novelty of the D-brane approach is that it tells us exactly how to calculate amplitudes for transition between open-string horizon states as external closed string states are emitted, absorbed or scattered: they are the familiar disc amplitudes, responsible for coupling between open and closed strings, which were much studied in type-I string theory. This direct access to the dynamics of horizon states is what has been the missing ingredient in discussions of the black hole information loss problem.

In short, we have in our hands an extremely simple model black hole in which physically very nontrivial calculations of horizon quantum dynamics seem to reduce to concrete string theory calculations. This should permit us to see how and whether the speculative mechanisms for resolving black hole paradoxes get realized. Three topics one might mention are the membrane paradigm [30], complementarity [35] and the holographic principle [29]. It is clear that the open strings provide a description of the state of a membrane on the horizon. It is clear that, from the D-brane perspective, there is nothing for an infalling closed string to do, apart from reflecting back to the outside world, except become an open

string on the horizon. These horizon surface degrees of freedom must encode the state of ordinary strings that, in the usual spacetime picture, have fallen into the world on the other side of the horizon. To implement complementarity, there must be some non-local transformation between the horizon open strings and effective closed strings describing the measurements accessible to an infalling observer who has fallen inside the horizon. To explore all this, and to better understand how string theory resolves the black hole paradoxes, one urgent task is to provide a good estimate of loop corrections in this background of open strings. It would also be interesting to try to find the mapping between the usual spacetime description of geodesics crossing the horizon and the three-plet of discrete numbers (i, j, n) (i, j label the particular branes) that is used to index the open string D-brane states. It would be very nice to find such a simple, and explicitly calculable, black hole system in four dimensions, though the essential qualitative features are probably already displayed in the five dimensional solution.

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